Transient Mass or Heat Transfer in Ensembles of Drops or Bubbles Translating at Low Reynolds Numbers

B. GAL-OR and I. YARON

Technion
Israel Institute of Technology, Haifa, Israel

Recently Chao (1969) has presented an elegant analytical solution of the transient heat or mass transfer to a single droplet translating in an inviscid medium. In his work he has also indicated that the same technique may be conveniently adopted to estimate mass transfer to or from a droplet moving at slow viscous motion in the range of small Reynolds but high Peclet numbers.

In the present communication we extend Chao's analysis to ensembles of many drops or bubbles translating in pure or nonpure fluids under the conditions of slow viscous motion. We also show that for large diffusional or thermal relaxation times the present results reduce to steady state expressions which were previously derived by Waslo and Gal-Or (1971) and by Yaron and Gal-Or (1971).

We consider an ensemble of many unbounded uniform spherical particles in slow viscous motion (Gal-Or and Waslo, 1968; Yaron and Gal-Or, in press). To characterize the velocity fields we employ here the solution of Gal-Or and Waslo (1968), given by

$$V_r^d = \frac{3}{2}\frac{U_s}{W}\beta(1-\phi^{5/3})(1-\eta^2)\cos\theta, \qquad (1)$$

$$V_\theta^d = \frac{3}{2}\frac{U_s}{W}\beta(1-\phi^{5/3})(2\eta^2-1)\sin\theta, \qquad (2)$$

$$0 \leq \eta \leq 1,$$

$$V_r^c = \frac{3}{2}\frac{U_s}{W}\left(\eta^2\phi^{5/3} - Y + \frac{W}{\eta} - \frac{1}{\eta^3}\right)\cos\theta, \qquad (3)$$

$$V_\theta^c = \frac{3}{2}\frac{U_s}{W}\left(-\frac{1}{2\eta^3} - \frac{W}{2\eta} + Y - 2\eta^2\phi^{5/3}\right)\sin\theta, \qquad (4)$$

$$1 \leq \eta \leq \phi^{-1/3},$$

where

$$W = 3 + 2\beta + 2\phi^{5/3}(1-\beta) \qquad (5)$$

$$Y = 2 + 2\beta + \phi^{5/3}(3-2\beta) \qquad (6)$$

and

$$\beta = \frac{\mu^c}{\mu^d + \gamma} \qquad (7)$$

Expressions (1) to (4) have been derived on the basis of the statistical cell model (Gal-Or, 1970). They represent expected statistical velocity fields averaged over the whole ensemble rather than those liable to be measured around any given particle (Gal-Or, 1970; Yaron and Gal-Or, in press). Possible retarding effects of adventitious surfactant impurities upon the internal circulation within the particles are also taken into account by the viscosity parameter β, whose physical significance is reported elsewhere (Gal-Or and Waslo, 1968; Yaron and Gal-Or, in press).

Under the condition of high Peclet numbers, that is, for thin diffusional or thermal boundary layers ($y << a$), the following approximations are justified for Equations (1) to (4) within the boundary layers

$$-V_r^d = V_r^c \simeq -3\frac{U_s}{W}\beta(1-\phi^{5/3})\cos\theta\frac{|y|}{a} \qquad (8)$$

$$V_\theta^d = V_\theta^c \simeq \frac{3}{2}\frac{U_s}{W}\beta(1-\phi^{5/3})\sin\theta \qquad (9)$$

The above approximations are subject to the additional constraint $y/\beta a << 1$. Accordingly, the present analysis is mainly applicable to ensembles of drops or bubbles with appreciable internal circulation.

By employing now Equations (8) and (9), the time-dependent diffusional boundary layer equations* for each phase of a binary isothermal particulate system can be cast in the dimensionless form

$$\frac{\partial\Phi^\alpha}{\partial\tau} - \frac{3}{2}Pe\cos\theta\,\overset{*}{y}{}^\alpha\frac{\partial\Phi^\alpha}{\partial\overset{*}{y}{}^\alpha} + \frac{3}{4}Pe\sin\theta\frac{\partial\Phi^\alpha}{\partial\theta} = \frac{\partial^2\Phi^\alpha}{\partial\overset{*}{y}{}^{\alpha 2}} \qquad (10)$$

where

$$\Phi^d = \frac{c^d - mc_\delta{}^c}{c_\delta{}^d - mc_\delta{}^c} \qquad (11)$$

$$\Phi^c = \frac{c^c - c_\delta{}^c}{c_\delta{}^d - mc_\delta{}^c} \qquad (12)$$

$$\tau = D^c t/a^2 \qquad (13)$$

$$\overset{*}{y}{}^d = \frac{|y|}{a}\left(\frac{D^c}{D^d}\right)^{1/2} \qquad (14)$$

$$\overset{*}{y}{}^c = \frac{y}{a} \qquad (15)$$

$$Pe = \frac{2a}{D^c}\frac{U_s}{W}\beta(1-\phi^{5/3}) \qquad (16)$$

Equation (16) represents, therefore, a modified Peclet number for the translating ensemble. Equation (10) is subjected to the following initial and boundary conditions:

$$\Phi^c(\overset{*}{y}{}^c, \theta, 0) = 0 \qquad (17)$$

Correspondence concerning this note should be addressed to I. Yaron at the Department of Fluid, Thermal and Aerospace Sciences, Case Western Reserve University, Cleveland, Ohio.

* The formulation for heat transfer is identical and is, therefore, omitted here.

$$\Phi^c(\overset{*}{y}^d, \theta, 0) = 1 \tag{18}$$

$$\Phi^c(\delta^c, \theta, \tau) = 0 \tag{19}$$

$$\Phi^d(\delta^d, \theta, \tau) = 1 \tag{20}$$

$$\Phi^d(0, \theta, \tau) = m\Phi^c(0, \theta, \tau) \tag{21}$$

$$\left(\frac{D^c}{D^d}\right)^{1/2} \frac{\partial \Phi^c}{\partial \overset{*}{y}^c}(0, \theta, \tau) = \frac{\partial \Phi^d}{\partial \overset{*}{y}^d}(0, \theta, \tau) \tag{22}$$

and

$$\frac{\partial \Phi^\alpha}{\partial \theta}(0, \theta, \tau) = \frac{\partial \Phi^\alpha}{\partial \theta}(0, \pi/2, \tau) = 0 \tag{23}$$

Note that in the case of heat transfer the distribution coefficient $m = 1$.

The general time-dependent solution of Equation (10), as reported by Chao (1969), is

$$\Phi^c = \frac{1}{m + (D^c/D^d)^{1/2}} \operatorname{erfc} \frac{N^c}{2\sqrt{Z}}, \quad N^c \geq 0 \tag{24}$$

$$\Phi^d = 1 - \frac{(D^c/D^d)^{1/2}}{m + (D^c/D^d)^{1/2}} \operatorname{erfc} \frac{|N^d|}{2\sqrt{Z}}, \quad N^d \leq 0 \tag{25}$$

where, for our case, the effects of viscosity, particle concentration and degree of purity of the fluids have been included in the parameters

$$N^\alpha = \frac{1}{2}\sqrt{3Pe}\, \overset{*}{y}^\alpha \sin^2\theta \tag{26}$$

$$Z = (f - \cos\theta) - \frac{1}{3}(f^3 - \cos^3\theta) \tag{27}$$

and

$$f = \frac{1 - \dfrac{1 - \cos\theta}{1 + \cos\theta} \exp\left(-\dfrac{3}{2} Pe_\tau\right)}{1 + \dfrac{1 - \cos\theta}{1 + \cos\theta} \exp\left(-\dfrac{3}{2} Pe_\tau\right)} \tag{28}$$

The instantaneous local Nusselt number for transfer at the particle's surface in the continuous phase is now given by

$$Nu^c = \frac{(3Pe/\pi)^{1/2}}{m + (D^c/D^d)^{1/2}} \frac{\sin^2\theta}{\sqrt{Z}} \tag{29}$$

and the transient Nusselt number, averaged over the entire surface of a particle, by

$$\overline{Nu}^c = \frac{2(Pe/\pi)^{1/2}}{m + (D^c/D^d)^{1/2}} I \tag{30}$$

where

$$I = \frac{\sqrt{3}}{4} \int_0^\pi \frac{\sin^3\theta}{\left[(f - \cos\theta) - \dfrac{1}{3}(f^3 - \cos^3\theta)\right]^{1/2}} d\theta \tag{31}$$

For conditions approaching steady state, that is, for $3\dfrac{U_s}{W}\beta(1 - \phi^{5/3})\dfrac{t}{a} \to \infty$, Equation (30) reduces to

$$\overline{Nu}^c_{ss} = \frac{2(Pe/\pi)^{1/2}}{m + (D^c/D^d)^{1/2}} \tag{32}$$

Redefining the Peclet number on the basis of the ensemble velocity (Gal-Or and Waslo, 1968).

$$U_{ens} = \frac{3}{2}\frac{U_s}{W}(Y - W\phi^{1/3}) \tag{33}$$

and considering the case of controlling resistance to transfer in the continuous phase alone, reduces Equation (30) to our previously published result (Waslo and Gal-Or, 1971; Yaron and Gal-Or, 1971)

$$\overline{Nu}^c_{ss} = \frac{2}{\sqrt{\pi}}\left(\beta \frac{1 - \phi^{5/3}}{Y - W\phi^{1/3}}\right)^{1/2} Pe_{ens}{}^{1/2} \tag{34}$$

For drops or bubbles in slow viscous motion, Chao's criterion for approach to the steady state transfer conditions can, therefore, be modified to include both the effects of particle concentration and the retardation due to impurities, namely,

$$\left(\beta \frac{1 - \phi^{5/3}}{Y - W\phi^{1/3}}\right) Pe_{ens\tau} \sim 1 \div 2 \tag{35}$$

These results are of course subject to the assumption that the relaxation times required for the ensemble of drops or bubbles to attain their terminal velocities are much shorter than the proper relaxation times for heat and mass transfer within the diffusional and thermal boundary layers.

ACKNOWLEDGMENT

This work is supported by Grant No. 11-1196 from Stiftung Volkswagenwerk, Hannover, Germany. The authors are grateful to B. T. Chao for his comments.

NOTATION

a = radius of a typical particle in the ensemble
c = concentration (binary system)
D = binary diffusion coefficient
f = function defined by Equation (28)
h = average heat transfer coefficient
I = integral function defined by Equation (31)
k = average mass transfer coefficient
m = distribution coefficient
N = parameter defined by Equation (26)
t = time
U_{ens} = ensemble velocity defined by Equation (33)
U_s = Stokes' terminal velocity
V_r = radial component of velocity vector
V_θ = tangential component of velocity vector
W = function defined by Equation (5)
Y = function defined by Equation (6)
y = radial distance from surface of particle
$\overset{*}{y}$ = dimensionless radial distance defined by Equations (14) and (15)
Z = function defined by Equation (27)

Greek Letters

α = heat conductivity
β = viscosity ratio defined by Equation (7)
γ = interfacial retardation viscosity defined in Gal-Or and Waslo (1968)
η = dimensionless radius, $\eta = r/a$
θ = polar angle measured from front stagnation
μ = viscosity
τ = dimensionless time defined by Equation (13)
Φ = dimensionless concentration defined by Equations (11) and (12)
ϕ = volume fraction of dispersed phase

Dimensionless Numbers for Ensembles of Particles

$Nu \equiv 2ak/D$ or $2ah/\alpha$

$$Pe \equiv \frac{2a}{D^c}\frac{U_s}{W}\beta(1-\phi^{5/3})$$

$$Pe_{ens} \equiv 2aU_{ens}/D^c$$

Superscripts and Subscripts

c	refers to continuous phase
d	refers to dispersed phase
α	refers to phase α ($\alpha = c$ or d)
$-$	value averaged over entire surface
ss	= steady state value
δ	= at edge of boundary layer

LITERATURE CITED

Chao, B. T., "Transient Heat and Mass Transfer to a Translating Droplet," *J. Heat Transfer, Trans. ASME*, Ser. C, **91**, 273 (1969).

Gal-Or, B., and S. Waslo, "Hydrodynamics of an Ensemble of Drops or Bubbles in the Presence of Surfactants," *Chem. Eng. Sci.*, **23**, 1431 (1968).

Gal-Or, B., "On Motion of Bubbles or Drops," *Can. J. Chem. Eng.*, **48**, 526 (1970).

Waslo, S., and B. Gal-Or, "Boundary Layer Theory for Mass and Heat Transfer in Clouds of Moving Drops, Bubbles, or Solid Particles," *Chem. Eng. Sci.*, **26**, 829 (1971).

Yaron, I., and B. Gal-Or, "Convective Mass or Heat Transfer from Size-Distributed Drops, Bubbles, or Solid Particles," *Int. J. Heat Mass Transfer*, **14**, 727 (1971).

———., "On Viscous Flow and Effective Viscosity of Concentrated Suspensions and Emulsions: Effect of Particle Concentration and Surfactant Impurities," *Rheol. Acta*, (in press).

Manuscript received August 30, 1972; revision received October 13, 1972; note accepted October 13, 1972.

Axial Dispersion of Non-Newtonian Fluids in Porous Media

LARRY W. PAYNE and HARRY W. PARKER

Department of Chemical Engineering
Texas Tech University, Lubbock, Texas 79409

Mixing of liquids in the direction parallel to flow through porous media, usually termed axial dispersion, is a significant factor in regard to chromatography columns, packed bed reactors, and miscible displacement methods for the recovery of petroleum. For this reason axial dispersion rates have frequently been investigated, but practically all investigations have employed low viscosity Newtonian fluids such as water and light hydrocarbons. Only one publication (Wen and Yim, 1971) has reported data for axial dispersion of non-Newtonian fluids flowing in porous media, and they found their data for non-Newtonian fluids were correlated by a previous comprehensive correlation for axial dispersion of Newtonian fluids, prepared by Chung and Wen (1968). Their polymer solutions were not highly non-Newtonian, and further work was recommended with fluids exhibiting greater non-Newtonian characteristics. In this research, pseudoplastic fluids having a power law exponent as low as 0.6 were employed at very low flow rates to facilitate the observation of non-Newtonian effects on axial dispersion rates. Details of this study are reported in Payne (1971).

EXPERIMENT

The flow system used in this investigation was a vertically oriented glass bead pack, the properties of which are shown in Table 1. Glass beads of 470 micron nominal size were packed into the flow cell while vibrating the cell. To obtain a one dimensional flow situation necessary for axial dispersion measurements, liquid distributors were installed at each end of the flow cell.

Solutions of 0.5 and 0.9% Polyox WSR-301, a poly(ethylene oxide) water soluble resin manufactured by Union Carbide Corporation, were used as the non-Newtonian fluids in this study. The rheological properties of these solutions, measured with a capillary viscometer, are shown in Figure 1. Neither solution exhibited power law behavior over the range of shear rates investigated as evidenced by the curvature of the log-log plots of shear stress versus shear rate.

A 90% solution of glycerol having a viscosity of 57 cp was used as a Newtonian fluid for comparisons with the Polyox solutions.

A Dupont Pontamine Sky Blue 6BX dye at a concentration of 30 ppm in the polymer or glycerol solutions was used as the tracer in this investigation.

These studies were conducted by displacing an undyed solution from the bead pack with a dyed solution at a constant rate or visa versa. Vertical, downward flow was used in all displacements, and the entire apparatus except for the pump was thermostated in an air bath at 27°C. The effluent from the pack was collected in small samples by an automated fraction collector and then analyzed for tracer concentration with spectrophotometer. Typical breakthrough profiles for the glycerol and 0.9% Polyox solutions are shown in Figure 2.

DISCUSSIONS OF RESULTS

The major variable in the miscible displacements was the velocity of flow through the bead pack. Reynolds numbers based on the particle diameter and interstitial velocity ranged from 3×10^{-4} to 20×10^{-4} for the glycerol solution. The approximate Reynolds number range for the

TABLE 1. PROPERTIES OF BEAD PACK

Packed length	59.5 cm
Flow cell diameter	3.81 cm
Effective bead diameter	0.0355 cm (calculated)
	0.0374 cm (measured)
Standard deviation of bead diameter	0.0019 cm
Void volume	248 cm^3
Porosity	0.365
Permeability	1.07×10^{-6} cm^2